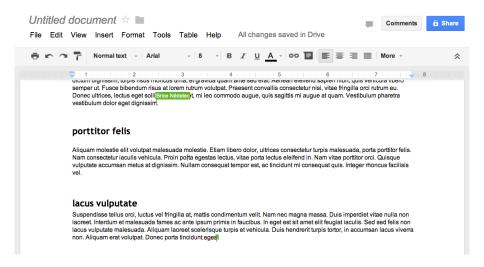
LSEQ: an Adaptive Distributed Sequence Data Structure

On the Fly Order Preserving Object Renaming

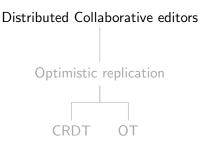
Achour Mostefaoui

joint work with Emmanuel Desmontils, Pascal Molli and Brice Nédelec





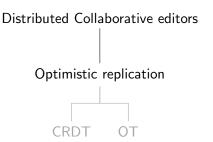
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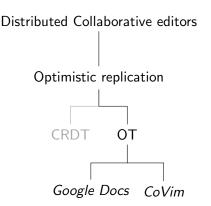
2 Two phases :

- a locally prepare operations to send
- **b** execute remote operations



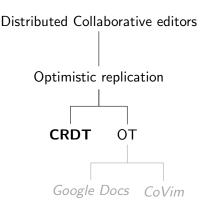
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- **3** Operational transform
 - + local operations cheap
 - remote operations complex



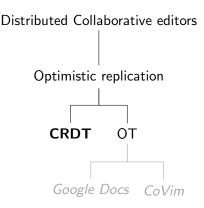
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• \nearrow collaborators \Rightarrow quadratic \nearrow remote operations

A document can be seen as a sequence od basic elements (characters, words, lines, etc.). The problem is non trivial because it is necessary that the edition (updating of the document) ensures the following three properties (CCI) :

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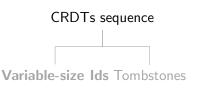
- **1** Convergence : the different copies need to converge to a same copy
- 2 Causality : any operation needs to reflect the operations that occurred causally before it
- 3 Intention : the effect of an operation needs to meet the intention of the user that ordered it

- **1** Two commutative operations :
 - Insert / delete
 - Identify the basic elements
 - The set of ids is totally ordered
 - The ids make the sequence



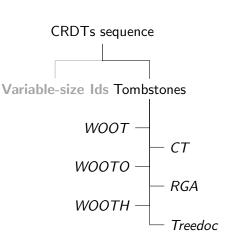
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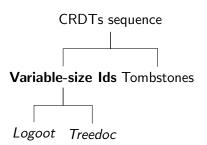
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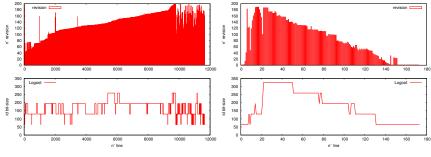
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- 4 The size of identifiers may grow
 - linearly wrt # operations



Motivations



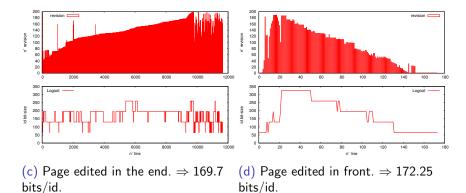


(a) Page edited in the end. \Rightarrow 169.7 bits/id.

(b) Page edited in front. \Rightarrow 172.25 bits/id.

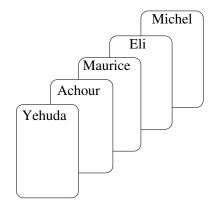
Motivations



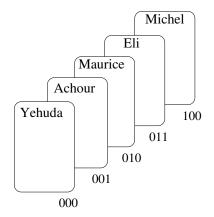


⇒ Allocation strategies are CRUCIAL

Abstract Problem (1)

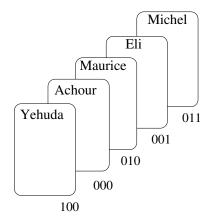


Abstract Problem (1)



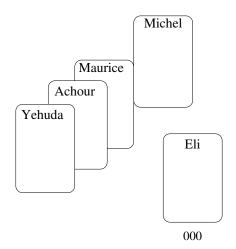
n cards can be named using ids of size $O(\log n)$

Abstract Problem (1)



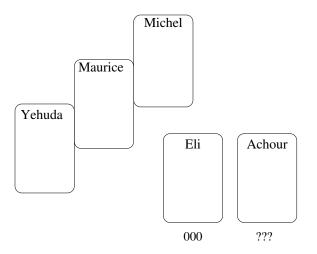
Even if one wants to preserve the order defined by the original names, n cards can be renamed with ids of size $O(\log n)$

Abstract Problem (2)



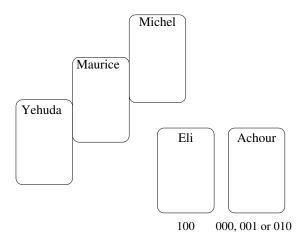
How about if the original names are not a priori known?

Abstract Problem (2)



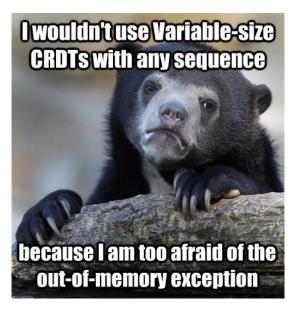
One needs to have spare space (dense set of ids)

Abstract Problem (2)

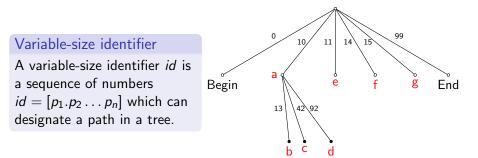


Is it possible to avoid all this loss of space?

Bear confesses...



Problem



Problem statement

Let \mathcal{D} a document on which *n* insert operations have been performed. Let $\mathcal{I}(\mathcal{D}) = \{id|(_, id) \in \mathcal{D}\}$. The function $alloc(id_p, id_q)$ should provide identifiers such as :

$$\sum_{d\in\mathcal{I}}\frac{|id|_2}{n} < O(n)$$

 $|id|_2$ means $log_2(id)$ aka. bit-length

Proposal : LSEQ

Three components :

- base doubling,
- multiple allocation strategies,
- random strategy choice.

Intuition

As it is complex to predict the editing behaviour, some depths of the tree on a given path can be lost if the reward **compensates** the loss. In other terms, even if LSEQ chooses the wrong strategy at a given time, it will eventually choose the good one, and that choice will **amortize** the cost of all previous lost depths.

Base doubling

Exponential trees :

- Under uniform distribution :
 - Spatial complexity : $O(n \log \log n)$. Where *n* the number of Ids.

$$[p_1.p_2...p_n] \Rightarrow |p_n|_2 = |p_{n-1}|_2 + 1$$
. Where $|p_1| = base$

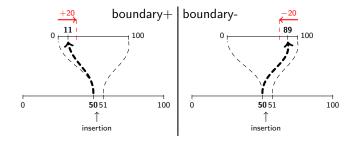
+1 bit \Rightarrow x2 identifiers

Intuition

If the number of insert operations is **low**, the id bit-length can stay **small**. On the other hand, when the number of insertions **increases**, it is profitable to allocate **larger** identifiers.

Multiple allocation strategies *boundary* :

- + Good : page edited in the end.
- Good : page edited in front.



Intuition

The allocation strategy *boundary* is **not sufficient** to be employed as a safe allocation strategy. However, by using its antagonist strategy, each strategy **cancels** the other's **deficiency**.

Random strategy choice

- Unique strategy : not sufficient
- \Rightarrow Strategy choice : When? Which?

Intuition : When

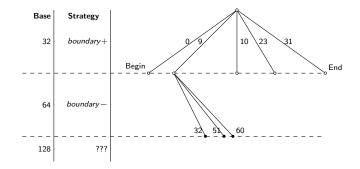
The **opening** of a new space has a major meaning : Either the allocation strategy went wrong, or, on the opposite, a high number of insertions saturated the previous depths, meaning that it requires more space. Therefore, the space opening is an ideal moment to decide which strategy to employ.

Intuition : Which

Since it is impossible to *a priori* know the editing behaviour, the strategy choice should **not favorize any behaviour**. Consequently, the frequency of appearence of each strategies must be equal.

Synthesis : example

- Exponential tree
- Two allocation strategies : *boundary*+ and *boundary*-
- Random strategy choice

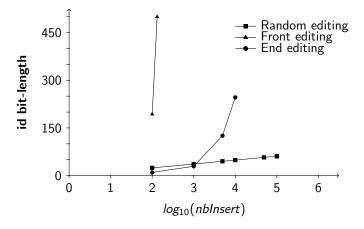


Experimentations

1 Influence of each LSEQ's component

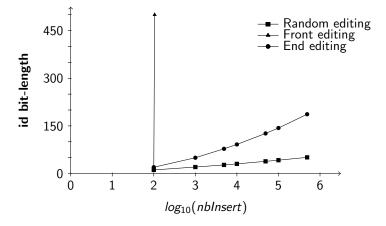
- \Rightarrow Synthetic documents.
- \Rightarrow High amount of insertions.
- \Rightarrow 3 editing behaviour : in the beginning, in the end, random.
- 2 Comparison with variable-size CRDT.
 - \Rightarrow Real documents : Wikipedia.
 - \Rightarrow 2 editing behaviour : in the beginning, in the end.

Boundary



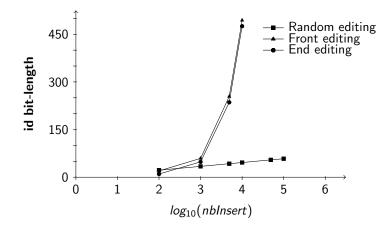
Simple *boundary*+ setup with $base = 2^{10}$ and *boundary* = 10

Exponential tree



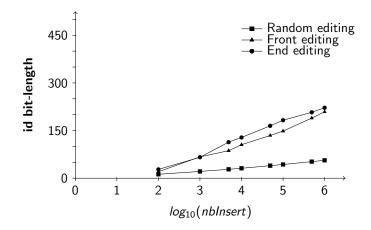
Base doubling setup with $base = 2^{4+id.size}$ and boundary = 10

Strategy choice



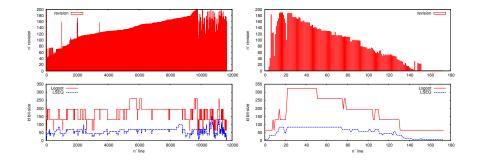
Round-Robin (**RR**) alternation of strategies *boundary*+ and *boundary*-(*base* = 2^{10} ; *boundary* = 10)

LSEQ



LSEQ randomly alternating boundary+ and boundary- and using the base doubling (base = $2^{4+id.size}$; boundary = 10)

Comparison with Logoot I



Comparison with Logoot II

		L	LSEQ
id-length	avg	2.65	6.25
	max	4	12
id-bit-length	avg	169.7	61.24
	max	256	150

Numerical values of a page edited in the end.

		L	LSEQ
id-length	avg	2.69	5.29
	max	5	8
id-bit-length	avg	172.25	51.99
	max	320	84

Numerical values on front edited page.

Synthesis : experiments

1 Each component contributes to LSEQ :

- Exponential tree : sub-linear behaviour
- Multiple strategies + choice : generic
- **2** Better than Logoot :
 - On documents edited in the end
 - On documents edited in the beginning

Conclusion and Future Works

Proof : sub-linear space complexity.

- *n* operations : uniform distribution $\Rightarrow O(\log n)$
- *n* operations : monotononic $\Rightarrow O((\log n)^2)$
- *n* operations : worst-case $\Rightarrow O(n^2)$???
- Proof : worst-case happens with a negligible probability

Concurrency effect