HOKKAIDO
U N I V E R S I T Y

# Arrangements of Hyperplanes 

## International Symposium

Bremen, December 1-3, 2014


# Hokkaido University - University of Bremen Joint Seminar 

# Arrangements of Hyperplanes 

December 1-3, 2014<br>ALTA, Universität Bremen, Bremen, Germany

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# Monday, December 1st, 2014 

## 14:00 Hiroaki Terao

Hokkaido University

## Free filtrations of affine arrangements of infinitely many hyperplanes

Consider an affine arrangement $\mathcal{A}$ consisting of infinitely many hyperplanes. An increasing sequence

$$
\mathcal{A}_{1} \subset \mathcal{A}_{2} \subset \cdots \subset \mathcal{A}_{i} \subset \ldots
$$

is called a free filtration if
(1) $\left|\mathcal{A}_{i}\right|=i$ and $\mathcal{A}_{i}$ is a free arrangement for each $i$, and
(2) $\bigcup_{i=1}^{\infty} \mathcal{A}_{i}=\mathcal{A}$.

In this talk, we will discuss the following problem:
Problem. If a free filtration exists, study the infinite sequence of the exponents $\left\{\exp \left(\mathcal{A}_{i}\right)\right\}_{i=1}^{\infty}$. Especially study the limit point of the infinite sequence of the points $\left[\exp \left(\mathcal{A}_{i}\right)\right]_{i=1}^{\infty}$ in the projective space. (Here the notation [ ] stands for the homogeneous coordinates.)

A seminal example is when $\mathcal{A}$ is an affine Weyl arrangement and each $\mathcal{A}_{i}$ is an ideal-Shi arrangement.

## 15:30 Michael Cuntz

## Leibniz Universität Hannover

## Triangles in arrangements of lines

Arrangements of lines which triangulate the plane, the so-called simplicial arrangements, appear to be rare. Many of them have been found during the last 70 years and it is conjectured that they are all known. In my talk I will present some old and some very recent observations, results, and conjectures on simplicial arrangements as well as several applications to these results.

## 16:30 Torsten Hoge <br> Leibniz Universität Hannover

## Nice arrangements

An arrangement is called nice if there is a partition of the hyperplanes which fulfills the singleton condition and the independence condition. This talk gives an overview of the joint work with Gerhard Röhrle on nice arrangements.

# Tuesday, December 2nd, 2014 

## 9:00 Gerhard Röhrle <br> Ruhr-Universität Bochum

## Inductive and recursive freeness of localizations of multiarrangements

The class of free arrangements plays a pivotal role in the study of hyperplane arrangements. While an arbitrary subarrangement of a free arrangement need not be free, freeness is retained by the class of subarrangements consisting of localizations. We extend this property to other classes of free arrangements, such as inductively free and recursively free arrangements. Indeed, we show this compatibility property for the corresponding more general classes of multiarrangements. This is a report on recent joint work with T. Hoge and A. Schauenberg.

## 10:30 Xia Liao <br> Technische Universität Kaiserslautern

## Free divisors with normal crossings

Normal crossing divisors is a fundamental notion in algebraic geometry. However, the definition of normal crossing divisors requires a choice of local analytic coordinates, and a purely algebraic definition is still missing. An algebraic description of normal crossing divisors is contained in Faber's conjecture: a free divisor is normal crossing if and only if the Jacobian ideal of the divisor is reduced. In this talk, we give an account of miscellaneous approaches to this conjecture in various cases. The best known case of the conjecture is when the divisor is locally quasi-homogeneous, of which we will indicate a proof at the end.

## 11:30 Elia Saini

## Université de Fribourg

## Ranks of Tutte groups of matroids

The inner Tutte group $\mathbb{T}_{M}^{(0)}$ of a matroid $M$ is a finitely generated abelian group introduced by Wenzel and Dress as algebraic counterpart to Tutte theory of matroids.
Using some topological properties of the space $\mathcal{R}_{\mathbb{C}}(M)$ of phasing classes of phased matroids with underlying matroid $M$, we give an explicit formula for the rank of $\mathbb{T}_{M}^{(0)}$. This, together with a result of Wenzel, completely determines the group $\mathbb{T}_{M}^{(0)}$ for each matroid with up to 7 elements.

## 14:30 Emanuele Delucchi

Université de Fribourg

## Toric arrangements - towards setting up a combinatorial theory

Recent work of De Concini, Procesi and Vergne on vector partition functions gave a new impulse to the study of toric arrangements from an algebraic, topological and combinatorial point of view. In this context, many new discrete structures have appeared in the literature, each describing some aspect of the theory (i.e., either the arithmetic-algebraic one or the topological one) and trying to mirror the combinatorial framework which revolves around arrangements of hyperplanes. I will give a quick overview of the state of the art and, taking inspiration from some recent topological results, I will try to suggest a possible approach towards unifying these different objects.

## 16:00 Filippo Callegaro

Università di Pisa

## Cohomology ring for toric arrangements

The topic of this talk is the integer cohomology ring of the complement of a real complexified toric arrangement. We will recall some basic combinatorial invariants and we will show how these can help to give a presentation of the toric analogous of the Orlik-Solomon algebra. One of the main techniques involved is the Leray spectral sequence. In the case of a non-unimodular arrangement, it is still an open problem to find a suitable combinatorial object that can determine the integer cohomology ring. This is joint work with E. Delucchi.

# Wednesday, December 3rd, 2014 

9:00 Takuro Abe<br>Kyoto University

## Freeness and geometry of line arrangements

Recently there are several progresses on free line arrangements in the projective plane. In this talk, two sufficient conditions for line arrangements to be free are given. They are related to the roots of characteristic polynomials and Betti numbers of line arrangements.

## 10:30 Michele Torielli

Università di Torino

## On the homotopy type and Orlik-Solomon algebra of supersolvable arrangements

In this talk we will present a new description of the minimal CW-complex of the complement of a supersolvable arrangement. This is a joint work with S. Settepanella.

## 13:00 Ivan Martino <br> Université de Fribourg

## Subspace arrangements and the Noether problem

We study subspace arrangements given by finite group actions. We show that they are connected to the Noether problem and to some new invariants recently introduced by Torsten Ekedahl.

## 14:30 Giovanni Gaiffi <br> Università di Pisa

## Group actions on wonderful models of braid arrangements

We will discuss some extended symmetric group actions that appear in the study of models of the braid arrangement and of the arrangements associated to complex reflection groups. Part of this is a joint work with F. Callegaro.

## List of participants

| Takuro Abe | Kyoto University |
| :--- | :--- |
| Roman Bruckner | Universität Bremen |
| Martin Burger | Universität Bremen |
| Filippo Callegaro | Università di Pisa |
| Michael Cuntz | Leibniz Universität Hannover |
| Emanuele Delucchi | Université de Fribourg |
| Eva-Maria Feichtner | Universität Bremen |
| Dmitry Feichtner-Kozlov | Universität Bremen |
| Giovanni Gaiffi | Università di Pisa |
| Tim Haga | Universität Bremen |
| Torsten Hoge | Leibniz Universität Hannover |
| Damien Imbs | Universität Bremen |
| Lukas Kühne | Universität Bonn |
| Xia Liao | Technische Universität Kaiserslautern |
| Tim Lindemann | Universität Bremen |
| Jan-Philipp Litza | Universität Bremen |
| Ivan Martino | Université de Fribourg |
| Paul Mücksch | Leibniz Universität Hannover |
| Viktoriya Ozornova | Universität Bremen |
| Christoph Pegel | Universität Bremen |
| Gerhard Röhrle | Ruhr-Universität Bochum |
| Elia Saini | Université de Fribourg |
| Kirsten Schmitz | Universität Bremen |
| Jan Senge | Universität Bremen |
| Simona Settepanella | Hokkaido University |
| Demet Taylan | Universität Bremen |
| Hiroaki Terao | Hokkaido University |
| Michele Torielli | Università di Torino |
|  |  |

## Organizers:

| Eva-Maria Feichtner | Simona Settepanella |
| :---: | :---: |
| Universität Bremen | Hokkaido University |
| Germany | Japan |

## Local information

## Workshop venue

Talks will be held at MZH 7200, on the 7th floor of the MZH building.

## Christmas market and workshop dinner

On Tuesday after the talks, we will go visit the Bremen Christmas market. The workshop dinner will take place afterwards in the city center, at the restaurant "Hofbräuhaus" at 8 pm .

## Wireless access

WLAN is accessible all over campus via eduroam. If you don't have an eduroam account, see the info sheet with login information to your personal WLAN guest account at University of Bremen.

## ATM

An ATM accepting EC- and major credit cards is located in a self-service branch of "Sparkasse" (red signs) right across the main entrance of the Mensa on the west-end of the Boulevard.


## Food on Campus

The Mensa (west-end of the boulevard) serves a large variety of warm food for lunch, 11:30-14:00.

Café (GW2) provides a variety of warm and cold snacks and also runs a coffee bar (upper floor) with a choice of coffee specialities (7:45-18:45, coffee bar: 11:30-16:30).

Café Unique (SFG) is a modestly priced campus restaurant. It is open 8:00-18:00, warm food is available 11:30-14:30.

Café Boulevard (west-end of the Boulevard) serves a variety of cold and warm snacks throughout the day, 8:00-17:00.

There is a small convenience store The FIZZ to go on Universitätsallee right across the streetcar stop for your daily needs, 7:00-21:00.

## Restaurants in Town

LA FATTORIA Sidewalk cafe and deli, excellent wine and cheese, $>20$ Euro, located between the university and the city centre on streetcar line 6, stop "Brahmsstraße."

ÜBERSEEMUSEUM Italian restaurant, about 15-25 Euro, inside the museum across from the main station.

LA DOLCE VITA Fancy Italian restaurant, $>20$ Euro, also close to the main station.
DONG FANG ASIA BUFFET Chinese buffet, high quality "eat all you can", 10 Euro dinner/7 Euro lunch, at about a 5 min walk from the staton in direction downtown.

MAREDO Steakhouse, about 25 Euro, two restaurants in the inner city.

KNURRHAHN Old style, German fish-restaurant, about 15-25 Euro.
FRIESENHOF North-german food, thus a lot of fish, about 20 Euro.
STÄNDIGE VERTRETUNG Restaurant themed around Rhenish culture, carnival and Rhenish specialities.

SCHLACHTE Riverside promenade along Weser with a huge variety of restaurants.


